

# Spin interference effects in ring conductors subject to Rashba coupling

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Quantum interference effects in rings provide suitable means for controlling spin at mesoscopic scales. Here we apply such control mechanisms to coherent spin-dependent transport in one- and two-dimensional rings subject to Rashba spin-orbit coupling. We first study the spin-induced modulation of unpolarized currents as a function of the Rashba coupling strength. The results suggest the possibility of all-electrical spintronic devices. Moreover, we find signatures of Berry phases in the conductance previously unnoticed. Second, we show that the polarization direction of initially polarized, transmitted spins can be tuned via an additional small magnetic control flux. In particular, this enables to precisely reverse the polarization direction at half a flux quantum. We present full numerical calculations for realistic two-dimensional ballistic microstructures and explain our findings in a simple analytical model for one-dimensional rings.

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## I. INTRODUCTION

In the last decade the field of *quantum electronics*<sup>1,2</sup> has received extraordinary attention from both experimental and theoretical physics communities. Special effort has been made towards control and engineering of the spin degree of freedom at the mesoscopic scale, usually referred to as *spintronics*.<sup>3,4</sup> The major problem faced in this field is the generation of spin-polarized carriers and their appropriate manipulation in a controllable environment, preferable in semiconductors. Since the original proposal of the spin field effect transistor by Datta and Das,<sup>5</sup> significant progress has been made<sup>6</sup> though the realization of a spin transistor still remains as a challenge. Setups based on intrinsic spin-dependent properties of semiconductors, as the Rashba effect<sup>7,8</sup> for a two-dimensional electron gas (2DEG) confined to an asymmetric potential well, appear to be of particular interest owing to the convenient means of all-electrical control through additional gate voltages.<sup>9</sup> In addition, coherent ring conductors enable to exploit the distinct interference effects of electron spin *and* charge which arise in these doubly connected geometries. This opens up the area of spin-dependent Aharonov-Bohm physics, including topics such as Berry phases,<sup>10,11</sup> spin-related conductance modulation,<sup>12,13</sup> persistent currents,<sup>14,15</sup> spin filters<sup>16</sup> and detectors,<sup>17</sup> spin rotation,<sup>18,19</sup> and spin switching mechanisms<sup>20,21,22</sup>.

In this article we focus on two different aspects of spin-interference in ballistic one- and two-dimensional (1D and 2D) ring geometries subject to Rashba spin-orbit coupling.<sup>29</sup> First, motivated by the work of Nitta et al.,<sup>12</sup> in Sec. II we revisit the subject of spin-induced modulation of unpolarized currents using the Hamiltonian for 1D rings recently introduced by Meijer et al.,<sup>32</sup> which slightly differs from the one used previously.<sup>12,24,33</sup> Taking into account the corresponding appropriate eigen-

states, we derive in Sec. III the modulation profile of the conductance as a function of the Rashba coupling strength and extract distinct effects due to the presence of Berry phases which have not been recognized in earlier work.<sup>12</sup> The 1D results are later compared with independent fully numerical calculations for 2D rings. The imprints of the Rashba coupling (strength) on the overall conductance is remarkable, pointing towards the possibility of all-electrical spintronics devices.

Second, and motivated by our previous work on spin control in the presence of external inhomogeneous magnetic fields,<sup>20</sup> we study in Sec. IV the magneto conductance of initially spin-polarized carriers traversing a ring geometry with Rashba spin-orbit interaction. We demonstrate by means of numerical calculations for 2D ring systems that the spin orientation of polarized carriers can be tuned and even reversed by means of an additional small magnetic control field. This implies a spin-switching mechanism which is probably more convenient for experimental realizations than our previous proposal,<sup>20</sup> since the originally suggested external inhomogeneous magnetic field is now replaced by the intrinsic effective field due to the Rashba interaction.

After a short summary in Sec. V we present details of our analytical approach in an Appendix.

## II. MODEL AND RELEVANT PARAMETERS

### A. Hamiltonian

The 2D quantum Hamiltonian for particles of charge  $-e$  ( $e > 0$ ) and effective mass  $m^*$  subject to Zeeman and Rashba coupling with coupling constants  $\mu$  and  $\alpha_R$ , respectively, reads

$$H_{2D} = \frac{1}{2m^*} \Pi^2 + \mu \mathbf{B} \cdot \boldsymbol{\sigma} + \frac{\alpha_R}{\hbar} (\boldsymbol{\sigma} \times \boldsymbol{\Pi})_z + V(\mathbf{r}), \quad (1)$$

where  $\boldsymbol{\sigma}$  is the vector of the Pauli spin matrices,  $\boldsymbol{\Pi} = \mathbf{p} + (e/c)\mathbf{A}$ , and  $\mathbf{B} = \nabla \times \mathbf{A}$ . The electrostatic potential  $V(\mathbf{r})$  defines, e.g., the confining potential of a 2D ballistic conductor. Recently it has been shown<sup>32</sup> that taking the limit from 2D to 1D rings (Fig. 1(a)) has to be performed by carefully considering in the above Hamiltonian (1) the radial wave functions in the presence of a narrow confinement. As a consequence, the corresponding 1D Hamiltonian for a ring of radius  $r_0$  in the presence of a vertical magnetic field  $\mathbf{B} = (0, 0, B)$  reads<sup>32,34</sup>

$$H_{1D} = \frac{\hbar\omega_0}{2} \left( -i\frac{\partial}{\partial\varphi} + \frac{\phi}{\phi_0} \right)^2 + \frac{\hbar\omega_B}{2} \sigma_z + \frac{\hbar\omega_R}{2} (\cos\varphi \sigma_x + \sin\varphi \sigma_y) \left( -i\frac{\partial}{\partial\varphi} + \frac{\phi}{\phi_0} \right) - i\frac{\hbar\omega_R}{4} (\cos\varphi \sigma_y - \sin\varphi \sigma_x), \quad (2)$$

where we have introduced the polar angle  $\varphi$ , the frequencies  $\omega_0 = \hbar/(m^*r_0^2)$ ,  $\omega_B = 2\mu B/\hbar$  and  $\omega_R = 2\alpha_R/(\hbar r_0)$ , and the magnetic fluxes  $\phi = \pi r_0^2 B$  and  $\phi_0 = hc/e$ .

The 1D eigenstates of (2) have the general form

$$\Psi_{\lambda,n}^s(\varphi) = \exp(i\lambda n\varphi) \chi_{\lambda,n}^s \quad ; \quad \chi_{\lambda,n}^s = \begin{pmatrix} \chi_1 \\ \chi_2 e^{i\varphi} \end{pmatrix}. \quad (3)$$

Here, the spin components  $\chi_{1,2}$  depend in principle on the travel direction  $\lambda = \pm 1$ , orbital quantum number  $n \geq 0$  ( $n$  integer), and spin  $s = \pm 1$ . The spin carriers being subject to  $H_{1D}$  experience an effective magnetic field  $\mathbf{B}_{\text{eff}} = \mathbf{B} + \mathbf{B}_R$  composed of the external field  $\mathbf{B}$  and the momentum dependent field  $\mathbf{B}_R$  arising from the Rashba coupling.  $\mathbf{B}_R$  lies in the plane of the ring.  $\mathbf{B}_{\text{eff}}$  encloses a tilt angle  $\alpha$  with the  $z$ -axis given by  $\tan\alpha = B_R/B = \omega_R(n' + 1/2)/\omega_B$  with  $n' = \lambda n + \phi/\phi_0$  (see Appendix A for further details). The exact orientation of  $\mathbf{B}_R$  is determined by the magnitude and sign of the momentum, namely  $\lambda n$ , i.e. spins travelling in opposite directions are subject to a different  $\mathbf{B}_R$ . Moreover, Eq. (2) implies that the orientation of  $\mathbf{B}_{\text{eff}}$  varies spatially.<sup>35</sup> This means that, in general, the corresponding spin eigenstates (3) are *not aligned* with  $\mathbf{B}_{\text{eff}}$  (see Fig. 1(b)). On the contrary, they are characterized by a different tilt angle  $\gamma$  determined by the relative magnitude of the spinor components  $\chi_1$  and  $\chi_2$ . However, in the limit of strong spin-orbit coupling, the so-called *adiabatic* regime, the spin eigenstates follow the local direction of the effective field, and  $\gamma \rightarrow \alpha$  (leading to Berry phases<sup>10</sup>). This limit is reached if the adiabaticity parameter  $Q = Q_B + Q_R$  satisfies  $Q \gg 1$ ,<sup>23,24</sup> where we have defined  $Q_B = \omega_B/(\omega_0|n' + 1/2|)$  and  $Q_R$ , particularly relevant here, as

$$Q_R = \omega_R/\omega_0. \quad (4)$$

Hence, the adiabatic limit corresponds to the situation where a spin precesses many times during a full travel around the ring.

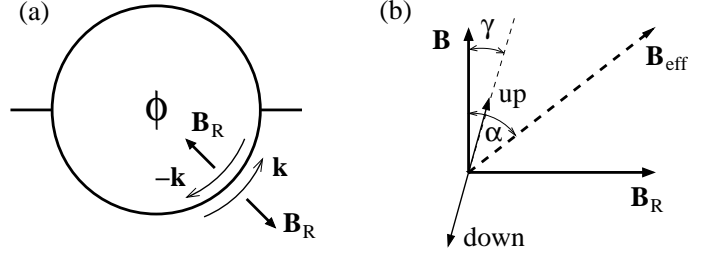


FIG. 1: (a) 1D ring of radius  $r_0$  subject to Rashba coupling in the presence of an additional, vertical magnetic field  $\mathbf{B}$  (flux  $\phi = \pi r_0^2 B$ ). Spin carriers travelling around the ring see a momentum ( $\mathbf{k}$ ) dependent in-plane Rashba field  $\mathbf{B}_R$ , which is orientationally inhomogeneous. (b) Up and down spin-eigenstates do not generally align with the total effective field  $\mathbf{B}_{\text{eff}} = \mathbf{B} + \mathbf{B}_R$ .

### B. 1D eigenstates in the absence of an external magnetic field

For  $\mathbf{B} = 0$  we have  $\omega_B = 0$  and  $\phi = 0$  in Eq. (2), and the Hamiltonian  $H_{1D}$  simplifies considerably. The resulting effective field reduces to the in-plane field  $\mathbf{B}_{\text{eff}} = \mathbf{B}_R$  with tilt angle  $\alpha = \pi/2$ . In this situation, the 1D eigenstates (3) take the simple form (see Appendix A for details)

$$\Psi_{+,n}^\uparrow(\varphi) = \exp(in\varphi) \begin{pmatrix} \sin\gamma/2 \\ \cos\gamma/2 e^{i\varphi} \end{pmatrix}, \quad (5)$$

$$\Psi_{+,n}^\downarrow(\varphi) = \exp(in\varphi) \begin{pmatrix} \cos\gamma/2 \\ -\sin\gamma/2 e^{i\varphi} \end{pmatrix}, \quad (6)$$

$$\Psi_{-,n}^\uparrow(\varphi) = \exp(-in\varphi) \begin{pmatrix} \cos\gamma/2 \\ -\sin\gamma/2 e^{i\varphi} \end{pmatrix}, \quad (7)$$

$$\Psi_{-,n}^\downarrow(\varphi) = \exp(-in\varphi) \begin{pmatrix} \sin\gamma/2 \\ \cos\gamma/2 e^{i\varphi} \end{pmatrix}. \quad (8)$$

The corresponding tilt angle  $\gamma$  is given by  $\tan\gamma = Q_R$ , satisfying  $\gamma \rightarrow \alpha = \pi/2$  in the adiabatic limit  $Q_R \rightarrow \infty$ . Hence, we note that the spinors  $\chi_{\lambda,n}^s$  in (5)-(8) do not actually depend on  $n$ . Moreover, the associated eigenenergies read

$$E_{\lambda,n}^s = \frac{\hbar\omega_0}{2} \left[ \left( \lambda n + \frac{1}{2} \right)^2 + \frac{1}{4} + s \left| \lambda n + \frac{1}{2} \right| \sqrt{1 + Q_R^2} \right]. \quad (9)$$

The above spin eigenstates (5)-(8) are defined in such a way that the eigenenergies (9) are maximum for spin-up states. We will make use of these results in the following section for the study of transport properties.

## III. RASHBA MODULATION OF UNPOLARIZED CURRENTS

We first consider the case where the 1D ring of Sec. II B is symmetrically coupled to two contact leads (Fig. 1(a))

in order to study the transport properties of the system subject to a constant, low bias voltage (linear regime). To this end we calculate the zero temperature conductance  $G$  based on the Landauer formula<sup>36</sup>

$$G = \frac{e^2}{h} \sum_{m', m=1}^M \sum_{\sigma', \sigma} T_{m'\sigma', m\sigma}^{\sigma'\sigma}, \quad (10)$$

where  $T_{m'\sigma', m\sigma}^{\sigma'\sigma}$  denotes the quantum probability of transmission between incoming  $(m, \sigma)$  and outgoing  $(m', \sigma')$  asymptotic states defined on semi-infinite ballistic leads. The labels  $m, m'$  and  $\sigma, \sigma'$  refer to the corresponding mode and spin quantum numbers, respectively. For 1D rings ( $M = 1$  in (10)) the transmission coefficients can be approximated to first order as follows: In the presence of Rashba coupling the energy splitting is such that particles with Fermi energy  $E_F$  can traverse the ring with four different wave numbers  $n_\lambda^s$ , depending on spin ( $s$ ) and direction of motion ( $\lambda$ ). The quantities  $n_\lambda^s$  are obtained by solving  $E_{\lambda,n}^s = E_F$  in Eq. (9) and do not require to be integer. Moreover, in this simple approach we assume perfect coupling between leads and ring (i.e. fully transparent contacts), neglecting backscattering effects leading to resonances. Thus, incoming spins  $|\sigma\rangle$  entering the ring at  $\varphi = 0$  propagate coherently along the four available channels and interfere at  $\varphi = \pi$ , leaving the ring in a mixed spin state  $|\sigma_{\text{out}}\rangle = \sum_{\lambda,s} \langle \chi_\lambda^s(0) | \sigma \rangle \exp(in_\lambda^s \pi) | \chi_\lambda^s(\pi) \rangle$ .<sup>37</sup> Choosing a complete basis of incoming and outgoing spin states, the spin-resolved transmission probabilities are obtained as  $T^{\sigma'\sigma} = |\langle \sigma' | \sigma_{\text{out}} \rangle|^2$ . After summation over the spin indices  $\sigma$  and  $\sigma'$ , we obtain for the total conductance

$$G = \frac{e^2}{h} \left[ 1 + \frac{1}{2} \left[ \cos \pi(n_-^\downarrow - n_+^\uparrow) + \cos \pi(n_-^\uparrow - n_+^\downarrow) \right] \right]. \quad (11)$$

Note that the phase difference acquired by opposite spin states travelling in opposite directions plays an important role for the modulation of the conductance.<sup>38</sup> The spin-dependent phases are signatures of the Aharonov-Casher effect<sup>39</sup> for spins travelling in the presence of an electric field, which is the electromagnetic dual of the Aharonov-Bohm effect.

By imposing  $E_{\lambda,n}^s = E_F$  in Eq. (9) we obtain

$$(n_-^\downarrow - n_+^\uparrow) = 1 + \sqrt{1 + Q_R^2}, \quad (12)$$

$$(n_-^\uparrow - n_+^\downarrow) = 1 - \sqrt{1 + Q_R^2}. \quad (13)$$

Inserting the above expressions into Eq. (11) one finds the total conductance as a function of the dimensionless Rashba coupling strength  $Q_R$ :

$$G = \frac{e^2}{h} \left\{ 1 + \cos \left[ \pi \left( \sqrt{1 + Q_R^2} - 1 \right) \right] \right\} \quad (14)$$

$$= \frac{e^2}{h} \{ 1 + \cos [\pi Q_R \sin \gamma - \pi(1 - \cos \gamma)] \}, \quad (15)$$

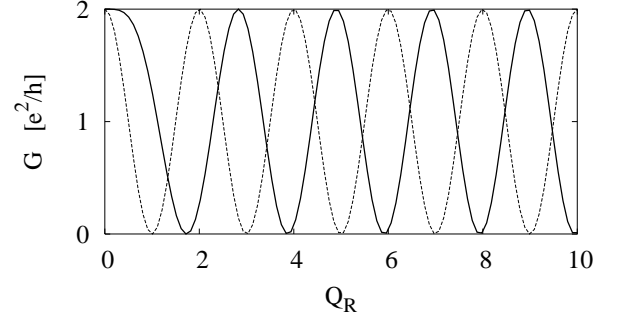


FIG. 2: Conductance modulation profile of 1D rings (Fig. 1(a)) as a function of the dimensionless Rashba strength  $Q_R$  in the absence of external magnetic field ( $\mathbf{B} = 0$ ). The curves show our result (14) for  $G$  (solid line) compared to the originally incomplete  $G_{\text{NMT}}$  of Eq. (16) (dashed line).

where we used  $\tan \gamma = Q_R$ ,  $\cos \gamma = 1/\sqrt{1 + Q_R^2}$ , and  $\sin \gamma = Q_R/\sqrt{1 + Q_R^2}$ . Comparing Eq. (14) with the corresponding result of Nitta, Meijer, and Takayanagi,<sup>12</sup>

$$G_{\text{NMT}} = \frac{e^2}{h} [1 + \cos(\pi Q_R)] , \quad (16)$$

we recognize two main contributions to the phase in (15): One is the Rashba phase  $\varphi_R = \pi Q_R \sin \gamma$ . This is similar to the phase  $\pi Q_R$ <sup>40</sup> appearing in  $G_{\text{NMT}}$ , Eq. (16), but corrected by a factor  $\sin \gamma$  accounting for the fact that the spinors are generally not aligned with  $\mathbf{B}_{\text{eff}}$ . In the limit of adiabatic spin transport both phases coincide (since  $\sin \gamma \rightarrow 1$  as  $Q_R \rightarrow \infty$ ). Moreover, we find an additional Aharonov-Anandan phase<sup>42</sup> contribution  $\varphi_{\text{AA}} = \pi(1 - \cos \gamma)$  to (15) absent in  $G_{\text{NMT}}$  and related to the solid angle accumulated by the change of spinor orientation during transport. In the adiabatic limit,  $\varphi_{\text{AA}}$  tends to the corresponding Berry phase  $\varphi_B = \pi(1 - \cos \alpha)$  as  $\cos \gamma \rightarrow \cos \alpha$  (where  $\cos \alpha = 0$ , i.e.  $\varphi_B = \pi$  in the present case).

In Fig. 2 we plot for comparison our result, Eq. (14), for  $G$  together with  $G_{\text{NMT}}$ , Eq. (16), as a function of the Rashba strength  $Q_R$ . There we observe that while  $G_{\text{NMT}}$  (dashed line) shows regular oscillations of period 2 in  $Q_R$ -units, our result (solid line) exhibits quasi-periodic oscillations of period larger than 2 reflecting the fact that non-adiabatic spin transport ( $\sin \gamma < 1$ ) takes place for small  $Q_R$ . For  $Q_R \gg 1$  the period is tending to 2 as the adiabatic limit is approached. In addition, a relative phase shift of magnitude  $\pi$  survives between  $G$  and  $G_{\text{NMT}}$  for large  $Q_R$ , coinciding with the appearance of the Berry phase  $\varphi_B = \pi$ . As a consequence, minima in  $G$  are obtained for even integers of  $\sqrt{Q_R^2 + 1}$ , i.e.  $Q_R = \sqrt{3}, \sqrt{15}, \dots$ . These minima are reminiscent of those found for the conductance of rings subject to Zeeman spin-coupling to in-plane circular magnetic fields (instead of Rashba coupling) as a function of the corresponding adiabaticity parameter.<sup>20</sup> Moreover, we note that Eq. (16), predicting uniform oscillations as a function of the coupling strength, actually corresponds to the conductance of a 1D ring subject to a radial electric field

of constant magnitude (instead of a vertical one as in the case of Rashba coupling).<sup>33,43</sup>

To complete the above discussion we present in the following the results of independent numerical calculations corresponding to more realistic 2D ring structures (Fig. 3). To this end we calculate the zero temperature conductance  $G$  based on the Landauer formula (10) by using a spin-dependent, recursive Green function technique<sup>44</sup> applied to the 2D Hamiltonian (1). Unless otherwise stated, our numerical calculations correspond to a quantum transmission averaged within a small energy window<sup>47</sup> in order to smooth out energy-dependent oscillations related to resonances in the ring structure. Moreover, the Rashba coupling is switched on and off adiabatically<sup>48</sup> within the leads by using a linear function.<sup>49</sup> Fig. 4 (solid line) shows the result for a single-mode ring of mean radius  $r_0$  and width  $w$  (aspect ratio  $w/r_0 \approx 0.3$ ) symmetrically coupled to two leads of the same width (see Fig. 3). This result is to be compared with that for the strictly 1D ring of Eq. (14) (dashed line; overall scaling factor included). Both curves present similar features on the whole. We observe that the first minimum of  $G$  in Fig. 4 coincides for both 1D and 2D calculations. However, as  $Q_R$  increases the 2D minima (solid line) undergo a small relative shift with respect to the 1D result (dashed line) and get less pronounced. This can be related to the finite aspect ratio of the ring: The strength  $Q_R$  can actually be written as  $Q_R = (r_0/w)\delta_R$ , where  $\delta_R = \alpha_R(2m^*/\hbar^2)w$  is the parameter defining the strength of the Rashba coupling in 2D conducting wires of width  $w$ .<sup>5,41</sup> The weak coupling regime characterized by spin subband separation is defined for  $\delta_R \ll 1$ . For the case of 1D wires and rings, this condition is always satisfied since  $w = 0$ . For finite width (represented by the finite  $w/r_0$  in our case) the situation is different, as we verify in our results of Fig. 4. There, the first minimum at  $Q_R = \sqrt{3}$  (fitting the above 1D result) corresponds to a relatively small coupling strength  $\delta_R \approx 0.5$ . However, at the second minimum,  $Q_R \approx 4 > \sqrt{15}$ , we already enter the strong coupling regime with  $\delta_R \approx 1.2$ . As a consequence deviations from the 1D case in the corresponding conductance modulation profile arise. This tendency is less pronounced as  $w/r_0 \rightarrow 0$  and the parameter  $\delta_R$  loses relevance.

Moreover, we note that in Fig. 4 the conductance minima of finite width rings (solid line) suffer the shift to *larger* values of  $Q_R$  with respect to the 1D results (dashed line) as  $Q_R$  increases. This suggests that the radial motion in 2D rings obstructs the approach to the regime of adiabatic spin transport, since a relatively larger coupling  $Q_R$  would be necessary for obtaining the same spinor tilt angle  $\gamma$  according to the structure of the phase (15).

Additionally, numerical results<sup>50</sup> not presented here indicate that the conductance of 2D ring structures supporting several open channels shows a modulation pattern similar to that of Fig. 4, provided that (i) the incoming and outgoing leads support just one open channel and (ii) the corresponding aspect ratio is small ( $w/r_0 \ll 1$ ).

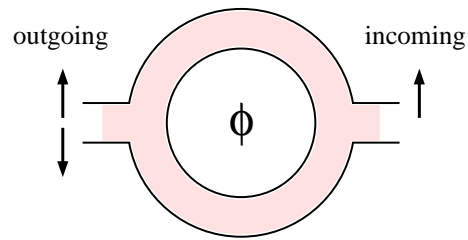


FIG. 3: 2D ring of mean radius  $r_0$  and width  $w$  used for numerical calculations of the conductance. The grey zone corresponds to the region subject to a finite Rashba coupling. This is switched on and off adiabatically within the leads by using a linear function. An additional, vertical magnetic field  $\mathbf{B}$  generates a flux  $\phi = \pi r_0^2 B$ .

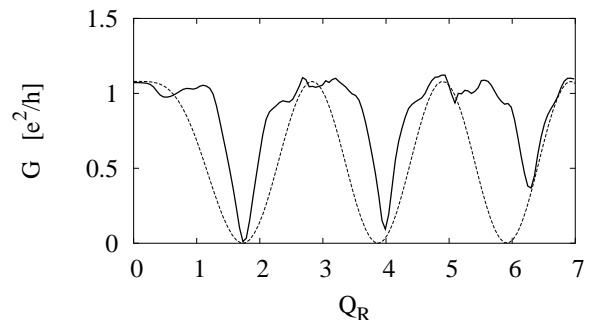


FIG. 4: Numerical calculation of the conductance modulation profile (solid line) of a single-mode 2D ring (Fig. 3, aspect ratio  $w/r_0 \approx 0.3$ ) as a function of the dimensionless Rashba strength  $Q_R$  in the absence of an external magnetic field ( $\mathbf{B} = 0$ ). Dashed line: corresponding 1D result, Eq. (14) (same as solid line in Fig. 2) including a fitting prefactor at  $Q_R = 0$  for comparison.

Furthermore, ring structures of irregular shape (leading to ballistic backscattering enhancement) exhibit a halving of the period in  $G(Q_R)$  modulation profiles when compared with that of Fig. 4, similarly to what is predicted for disordered systems.<sup>43,51,52</sup>

#### IV. MAGNETOCONDUCTANCE OF SPIN POLARIZED CURRENTS AND SPIN SWITCHING

In this section we discuss the possibility of controlling the spin orientation of *spin-polarized* carriers by means of distinct interference effects in mesoscopic ring structures due to (charge and spin) quantum coherence. Motivated by our previous work on spin-switching in the presence of in-plane circular magnetic fields<sup>20</sup> we study here the magnetoconductance of incoming spin-polarized carriers,<sup>53</sup> now subject to Rashba interaction.

The setup proposed is that of Fig. 3, representing a 2D ring (aspect ratio  $w/r_0 \approx 0.3$ ) subject to Rashba coupling symmetrically coupled to two leads. In addition, a weak



ring structures supporting several open modes show features similar to that of Fig. 5 for single-mode rings, as long as (i) the incoming and outgoing leads support just one open channel and (ii) the corresponding aspect ratio is small ( $w/r_0 \ll 1$ ). Deviations from e.g. Fig. 5(b) arise as  $w/r_0$  increases, manifested by a less defined minimum in  $G^{\uparrow\uparrow}$  at  $\phi = \phi_0/2$  due to the relatively large fraction of flux  $\phi$  penetrating the finite-width ring in that case. Moreover, asymmetric rings with arms of different effective length can also show a flux-modulated spin polarization similar to that of Fig. 5(b). However, the spin switching is not complete, and it does not necessarily take place at  $\phi = \phi_0/2$ .

Analytical results for the spin-switching in 1D rings can be, in principle, obtained by studying the spin resolved transmission probabilities  $T^{\sigma'\sigma}$  as defined in Sec. III.

## V. CONCLUSIONS

We have studied coherent spin-dependent transport in ballistic 1D and 2D ring geometries subject to (spin-orbit) Rashba coupling. We first obtained, via analytical (1D) and numerical (2D) calculations, the spin-related conductance modulation profile of *unpolarized* spin carriers as a function of the scaled Rashba strength  $Q_R$ , which also acts as a measure defining adiabatic spin transport for  $Q_R \gg 1$ . The conductance appears to be quite sensitive to  $Q_R$ , suggesting the possibility of all-electrical spintronic devices. Moreover, we point out the role played by Aharonov-Anandan and Berry phases unnoticed in a previous proposal.<sup>12</sup> In addition, we also studied the magneto conductance of *spin-polarized* carriers to assess possibilities for controlling the spin orientation in the presence of Rashba coupling. We demonstrate that an additional small flux  $\phi$  can be used as a control parameter for inducing spin flips. The mechanism arises from cooperative quantum interference of charge and spin degrees of freedom in coherent transport. Combined with a spin detector such a device may be used for controlling spin polarized currents alternative to the Datta-Das transistor.<sup>5</sup> Moreover, we note that the Dresselhaus spin-

orbit coupling,<sup>30</sup> not studied here, could lead to similar conductance-modulation and spin-switching effects. However, its interplay with the Rashba coupling in systems where both contributions are comparable can produce further effects of interest.<sup>31,55</sup>

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## APPENDIX A: 1D SPIN EIGENSTATES AND EFFECTIVE RASHBA FIELD

The components of the eigenstates  $\Psi_{\lambda,n}^s$  of the 1D Hamiltonian (2), which are given in Eq. (3) (spin  $s = \pm 1$ , travel direction  $\lambda = \pm 1$ , integer orbital number  $n \geq 0$ ), satisfy the matrix equation

$$\begin{pmatrix} \frac{\hbar\omega_0}{2}n'^2 + \frac{\hbar\omega_B}{2} & \frac{\hbar\omega_R}{2}\left(n' + \frac{1}{2}\right) \\ \frac{\hbar\omega_R}{2}\left(n' + \frac{1}{2}\right) & \frac{\hbar\omega_0}{2}(n'+1)^2 - \frac{\hbar\omega_B}{2} \end{pmatrix} \chi = E_{\lambda,n}^s \chi \quad (\text{A1})$$

where the normalized spinors read

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \frac{1}{\sqrt{1 + (\Delta_{\lambda,n}^s)^2}} \begin{pmatrix} 1 \\ \Delta_{\lambda,n}^s \end{pmatrix}, \quad (\text{A2})$$

with

$$\Delta_{\lambda,n}^s = \frac{E_{\lambda,n}^s - (\hbar\omega_0/2)n'^2}{(\hbar\omega_R/2)(n' + 1/2)}, \quad (\text{A3})$$

$n' = \lambda n + \phi/\phi_0$ , and eigenvalues given by

$$E_{\lambda,n}^s = \frac{\hbar\omega_0}{2} \left\{ \left[ \left( n' + \frac{1}{2} \right)^2 + \frac{1}{4} \right] + s \sqrt{\left[ \left( n' + \frac{1}{2} \right) - \frac{\omega_B}{\omega_0} \right]^2 + \left( \frac{\omega_R}{\omega_0} \right)^2 \left( n' + \frac{1}{2} \right)^2} \right\}. \quad (\text{A4})$$

The off-diagonal elements on the left-hand side of Eq. (A1) determine the magnitude and orientation of the in-plane effective Rashba field  $\mathbf{B}_R$ . The resulting overall effective field  $\mathbf{B}_{\text{eff}} = \mathbf{B} + \mathbf{B}_R$  has a tilt angle  $\alpha$  with respect to the  $z$ -axis satisfying  $\tan \alpha = \omega_R(n' + 1/2)/\omega_B$ .

Moreover, the presence of the kinetic terms in the diagonal elements of (A1) prevent the spinors  $\chi$  to align with  $\mathbf{B}_{\text{eff}}$ . Instead, they are characterized by a tilt angle  $\gamma$  which tends to  $\alpha$  only for strong spin coupling (adiabatic limit).

For illustration we discuss the spin-up case ( $s = 1$ ) and the dependence on the travel direction  $\lambda$  in the absence of Zeeman coupling ( $\omega_B = 0$ ,  $\alpha = \pi/2$ ) provided that a finite flux  $\phi$  is present. Then we find from (A2)

$$\chi_1^\uparrow = \frac{Q_R}{\sqrt{2}[Q_{n'} + Q_R^2]^{1/2}} \quad (\text{A5})$$

$$\chi_2^\uparrow = \frac{Q_{n'}}{\sqrt{2}[Q_{n'} + Q_R^2]^{1/2}}, \quad (\text{A6})$$

where  $Q_{n'} = 1 + \text{sign}[n' + 1/2]\sqrt{1 + Q_R^2}$ . The dimensionless Rashba strength  $Q_R$  is defined in Eq. (4). In the adiabatic, strong coupling limit ( $Q_R \gg 1$ ) we obtain from (A5) and (A6)

$$\chi^\uparrow \xrightarrow{Q_R \gg 1} \begin{cases} \begin{pmatrix} 1/\sqrt{2}^- \\ 1/\sqrt{2}^+ \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = 1 \\ \begin{pmatrix} 1/\sqrt{2}^+ \\ -1/\sqrt{2}^+ \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = -1 \end{cases} \quad (\text{A7})$$

indicating that the spinors are contained within the plane defined by the ring and pointing along  $\mathbf{B}_R$ . On the other

hand, in the opposite limit of weak coupling ( $Q_R \ll 1$ ), we arrive at

$$\chi^\uparrow \xrightarrow{Q_R \ll 1} \begin{cases} \begin{pmatrix} 0^+ \\ 1^- \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = 1 \\ \begin{pmatrix} 1^- \\ 0^- \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = -1 \end{cases} \quad (\text{A8})$$

highlighting the influence of the travelling direction on the relative orientation of the spinors. As a consequence we find that the up spinors can be written as

$$\chi^\uparrow = \begin{cases} \begin{pmatrix} \sin \gamma/2 \\ \cos \gamma/2 \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = 1 \\ \begin{pmatrix} \cos \gamma/2 \\ -\sin \gamma/2 \end{pmatrix} & \text{if } \text{sign}[n' + 1/2] = -1 \end{cases} \quad (\text{A9})$$

with  $\tan \gamma = Q_R$ . Following a similar procedure for  $\phi = 0$  ( $n' = \lambda n$ ) we find the eigenstates listed in Sec. II B.

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- $((-i\partial/\partial\varphi + \phi/\phi_0) \gg 1)$ .
- <sup>35</sup> See e.g. that for large angular momentum the effective Rashba field  $\mathbf{B}_R$  is determined by the second line of Eq. (2), corresponding to an effective radial field (i.e. (anti)parallel to  $\hat{\mathbf{r}} = \cos\varphi \hat{x} + \sin\varphi \hat{y}$ ).
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- <sup>48</sup> Note that here the term *adiabatic* is used in a sense different from the one introduced previously in the context of adiabatic spin transport.
- <sup>49</sup> The adiabatic switching of the Rashba coupling is introduced in order to avoid an abrupt transition between the region of interest subject to finite coupling (the ring) and the leads connected to reservoirs free of spin-orbit coupling. For a given energy, some differences can arise in the computed conductance when using or not the adiabatic switching in the case of strong Rashba coupling. However, no significant differences survive when performing an energy average in the way described above.
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- <sup>54</sup> Note the difference with respect to the Datta-Das proposal,<sup>5</sup> where incoming spin-polarized states point along the  $z$ - or  $x$ -axis to get spin oscillations.
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